ACOUSTIC STREAMING IN A BOUNDARY LAYER

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Many papers have been devoted to the analysis of acoustic streaming in boundary layers [1-7]. The acoustic streaming problem has been solved between parallel walls [1], in a tube [2], around circular elliptical cylinders [3, 4], and around an arbitrary curvilinear interface [5, 6]. In all these papers, however, the influence of the thermal conductivity of the fluid on the acoustic streaming is neglected (with one obvious exception [2]). Moreover, all the papers essentially treat the special case of streaming in a wave field contiguous with the boundary. In the present article we determine the acoustic streaming in a boundary layer in the field of an obliquely incident plane wave in a viscous heat-conducting medium.

Slow streaming in a boundary layer is described by the well-known equation [5, 7]

$$\rho_0\nu\Delta\mathbf{V}_2 - \mathbf{F} = 0, \qquad (1)$$
$$\mathbf{F} = -\rho_0\langle(\mathbf{V}_1\cdot\nabla)\mathbf{V}_1 + \mathbf{V}_1(\nabla\cdot\mathbf{V}_1)\rangle,$$

in which ρ_0 is the density of the fluid in the undisturbed state, $\mathbf{V}_2$ is the acoustic streaming velocity, ν is the kinematic viscosity coefficient, and the angle brackets denote time average.

The expression derived by Konstantinov [8] for the first-approximation acoustic field $\mathbf{V}_1$ can be written as follows for small grazing angles of the incident wave:

$$\mathbf{V}_1 = \mathbf{V}_{1a} + \mathbf{V}_{1\text{п}} + \mathbf{V}_{1\text{в}} + \mathbf{V}_{1\text{т}};$$
$$\begin{cases} \mathbf{V}_{1a} = k_{11}M(\mathbf{e}_x - \eta\mathbf{e}_z),\ \mathbf{V}_{1\text{п}} = k_{11}Mv_\text{п}e^{i\varphi_\text{п}}(\mathbf{e}_x + \eta\mathbf{e}_z), \\ \mathbf{V}_{1\text{в}} = M\omega_1\exp\left[i\left(\beta z + \varphi_\text{в} + \frac{\pi}{4}\right)\right](-\beta\sqrt{2}\mathbf{e}_x + k_{11}\mathbf{e}_z), \\ \mathbf{V}_{1\text{т}} = \alpha\sqrt{2}\,v_\text{т}\,\omega_2\exp\left[i\left(\alpha z + \varphi_\text{т} + \frac{\pi}{4}\right)\right]\mathbf{e}_z, \\ M = \exp\left[i\left(k_{11}x - \sigma t + \frac{\pi}{2}\right)\right],\ \omega_1 = \exp(-\beta z),\ \omega_2 = \exp(-\alpha z), \end{cases} \qquad (2)$$

where $\mathbf{V}_{1a}$, $\mathbf{V}_{1\Pi}$ are the velocities in the longitudinal incident and reflected waves, $\mathbf{V}_{1B}$ is the velocity in the viscosity wave, $\mathbf{V}_{1T}$ is the velocity in the thermal wave, $\mathbf{e}_x$, $\mathbf{e}_z$ are unit vectors of a Cartesian coordinate system with its axis Ox directed along the boundary, $k_{11}=\sigma/c$ is the longitudinal wave number, and β^{-1}, α^{-1} are the thickness scales of the viscous and thermal acoustic boundary layers.

The moduli ν_Π, ν_B, ν_T and phases φ_Π, φ_B, φ_T of the reflection coefficients are given by the expressions

$$v_\text{п} = \frac{\sqrt{4\eta^4 + \Phi^4}}{(\eta + \Phi)^2 + \eta^2},\quad \varphi_\text{п} = \operatorname{arctg}\frac{2\eta\Phi}{2\eta^2 - \Phi^2},$$
$$\Phi = (\gamma - 1)\frac{\sqrt{2\sigma\varkappa_\text{т}}}{c} + \frac{\sqrt{2\sigma\nu}}{c},\ v_\text{т} = (\gamma - 1)\lambda_2^2\sqrt{v_\text{п}^2 + 2v_\text{п}\cos\varphi_\text{п} + 1}, \qquad (3)$$
$$\varphi_\text{т} = \operatorname{arctg}\frac{v_\text{п}\sin\varphi_\text{п}}{1 + v_\text{п}\sin\varphi_\text{п}} - \frac{\pi}{2},\ v_\text{в} = \lambda_1\sqrt{v_\text{п}^2 + 2v_\text{п}\cos\varphi_\text{п} + 1},$$
$$\varphi_\text{в} = \varphi_\text{т} + \frac{\pi}{4},\ \lambda_1 = \frac{k_{11}}{\beta\sqrt{2}},\ \lambda_2 = \frac{k_{11}}{\alpha\sqrt{2}},$$

where γ is the adiabatic exponent, $\varkappa_T$ is the thermal diffusivity, and Φ is the critical angle of Konstantinov [8].

Beginning with the work of Schlichting [3], the theory of acoustic streaming near boundaries has been based on the boundary-layer equations. According to this theory, terms of order greater than $1/\beta l$ (where l is the space scale of the acoustic field) are retained in all the mathematical expressions. However, to obtain the dependence of the acoustic streaming velocity on the grazing angle of the incident wave the indicated approx-

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imation is inadequate. We therefore retain terms through order $1/\beta l$ inclusively in the first-approximation expressions for the acoustic field (3).

In a boundary layer the tangential component of the force $\mathbf{F}$ is known to be much larger than the normal component [5], i.e., we can assume that $\mathbf{F}=F_x\mathbf{e}_x$. In this case $\mathbf{V}_2$ is also directed along the axis Ox, and $\mathbf{V}_2=V_2(z)\mathbf{e}_x$. to facilitate the ensuing calculations in the case of multiple-wave processes it is practical to write Eq. (1) in matrix form. To do so we introduce the matrix of pair interactions

$$S=\begin{Vmatrix} F_{\text{аа}} & F_{\text{ап}} & F_{\text{ав}} & F_{\text{ат}} \\ F_{\text{па}} & F_{\text{пп}} & F_{\text{пв}} & F_{\text{пт}} \\ F_{\text{ва}} & F_{\text{вп}} & F_{\text{вв}} & F_{\text{вт}} \\ F_{\text{та}} & F_{\text{тп}} & F_{\text{тв}} & F_{\text{тт}} \end{Vmatrix}.$$

All except the diagonal elements of this matrix characterize the forces created by interaction of different wave modes. The diagonal elements, on the other hand, describe the forces created by self-action of the waves:

$$F_{\text{ап}}=-\rho_0\langle(\mathbf{V}_{1\text{а}}\cdot\nabla)V_{1\text{п}x}+V_{1\text{а}x}(\nabla\cdot\mathbf{V}_{1\text{п}})\rangle,$$
$$F_{\text{па}}=-\rho_0\langle(\mathbf{V}_{1\text{п}}\cdot\nabla)V_{1\text{а}x}+V_{1\text{п}x}(\nabla\cdot\mathbf{V}_{1\text{а}})\rangle,$$
$$F_{\text{аа}}=-\rho_0\langle(\mathbf{V}_{1\text{а}}\cdot\nabla)V_{1\text{а}x}+V_{1\text{а}x}(\nabla\cdot\mathbf{V}_{1\text{а}})\rangle.$$

With the introduction of the matrix S the acoustic streaming equation (1) now takes the form

$$\frac{d^2}{dt^2}\begin{Vmatrix} V_{2\text{а}} \\ V_{2\text{п}} \\ V_{2\text{в}} \\ V_{2\text{т}} \end{Vmatrix}=-(\rho_0\nu)^{-1}\|S\|\times\begin{Vmatrix}1\\1\\1\\1\end{Vmatrix}, \tag{4}$$
$$V_2=V_{2\text{а}}+V_{2\text{п}}+V_{2\text{в}}+V_{2\text{т}},$$

where V_{2a}, $V_{2\Pi}$, V_{2B}, and V_{2T} are the x components of the acoustic streaming velocities created by interaction of the corresponding wave with all other wave modes, including self-action.

Integrating Eq. (4) with the boundary conditions $V_2|_{z=0}=0,\ dV_2/dz|_{z\to\infty}=0$, we obtain

$$V_{2\text{а}}=\omega\left[\eta v_{\text{в}}(\omega_1\sin Q_{\text{в}}-\sin\varphi_{\text{в}})+v_{\text{т}}(\omega_2\sin Q_{\text{т}}-\sin\varphi_{\text{т}})\right], \tag{5}$$
$$V_{2\text{п}}=\omega v_{\text{п}}v_{\text{т}}\left[\omega_2\sin(Q_{\text{в}}-\varphi_{\text{п}})-\sin(\varphi_{\text{т}}-\varphi_{\text{п}})\right]-\omega v_{\text{п}}v_{\text{в}}\eta\left[\omega_1\sin(Q_{\text{в}}-\varphi_{\text{п}})-\right.$$
$$\left.-\sin(\varphi_{\text{в}}-\varphi_{\text{п}})\right],\ V_{2\text{т}}=\frac{\omega}{\lambda_2}v_{\text{т}}v_{\text{в}}\frac{1}{(\mathrm{Pr}+1)^2}\left\{\omega_1\omega_2\left[(1-\mathrm{Pr})\sin\left(Q_{\text{в}}-Q_{\text{т}}+\frac{3}{4}\pi\right)-\right.\right.$$
$$\left.-2\mathrm{Pr}^{1/2}\cos\left(Q_{\text{в}}-Q_{\text{т}}+\frac{3}{4}\pi\right)\right]+\left[2\mathrm{Pr}^{1/2}\cos\left(\varphi_{\text{в}}-\varphi_{\text{т}}+\frac{3}{4}\pi\right)-\right.$$
$$\left.\left.-(1-\mathrm{Pr})\sin\left(\varphi_{\text{в}}-\varphi_{\text{т}}+\frac{3}{4}\pi\right)\right]\right\},$$
$$V_{2\text{в}}=\omega v_{\text{в}}\left\{\lambda_1\left[\omega_1\sin\left(Q_{\text{в}}-\frac{\pi}{4}\right)-\sin\left(\varphi_{\text{в}}-\frac{\pi}{4}\right)+\lambda_1 v_{\text{п}}\left[\omega_1\sin\left(Q_{\text{в}}-\varphi_{\text{п}}-\right.\right.\right.\right.$$
$$\left.\left.-\frac{\pi}{4}\right)-\sin\left(\varphi_{\text{в}}-\varphi_{\text{п}}-\frac{\pi}{4}\right)\right]-\frac{1}{2}v_{\text{в}}(1-\omega_1^2)+\frac{v_{\text{т}}}{\lambda_1}\frac{1}{(1+\mathrm{Pr}^{-1})^2}\left[\omega_1\omega_2\left[(\mathrm{Pr}^{-1}-\right.\right.$$
$$\left.-1)\sin\left(Q_{\text{в}}-Q_{\text{т}}-\frac{3}{4}\pi\right)-2\mathrm{Pr}^{-1/2}\cos\left(Q_{\text{в}}-Q_{\text{т}}-\frac{3}{4}\pi\right)\right]+\left[2\mathrm{Pr}^{-1/2}\times\right.$$
$$\left.\left.\left.\times\cos\left(\varphi_{\text{в}}-\varphi_{\text{т}}-\frac{3}{4}\pi\right)-(\mathrm{Pr}^{-1}-1)\sin\left(\varphi_{\text{в}}-\varphi_{\text{т}}-\frac{3}{4}\pi\right)\right]\right]\right\},\ Q_{\text{в}}=\beta z+\varphi_{\text{в}},\ Q_{\text{т}}=$$
$$=\alpha z+\varphi_{\text{т}},\ \omega=\frac{A^2}{k_{11}\nu},$$

where A is the amplitude of the incident field and $\mathrm{Pr}=\nu/\varkappa_T$ is the Prandtl number.

The streaming velocities on the outer side of the boundary layer are determined from (5) as $z\to\infty$:

$$V_{2\text{а}}=-\omega\left[\eta v_{\text{в}}\sin\varphi_{\text{в}}+v_{\text{т}}\sin\varphi_{\text{т}}\right],\ V_{2\text{п}}=\omega\left[v_{\text{п}}v_{\text{т}}\sin(\varphi_{\text{п}}-\varphi_{\text{т}})+\right.$$
$$\left.+\eta v_{\text{п}}v_{\text{в}}\sin(\varphi_{\text{в}}-\varphi_{\text{п}})\right],\ V_{2\text{т}}=\frac{\omega}{\lambda_2}v_{\text{т}}v_{\text{в}}\frac{1}{(\mathrm{Pr}+1)^2}\left[2\mathrm{Pr}^{1/2}\cos\left(\varphi_{\text{в}}-\varphi_{\text{т}}+\frac{3}{4}\pi\right)-\right.$$
$$\left.-(1-\mathrm{Pr})\sin\left(\varphi_{\text{в}}-\varphi_{\text{т}}+\frac{3}{4}\pi\right)\right],\ V_{2\text{в}}=\omega v_{\text{в}}\left\{\lambda_1\sin\left(\frac{\pi}{4}-\varphi_{\text{в}}\right)-\right. \tag{6}$$
$$-\lambda_1 v_{\text{п}}\sin\left(\varphi_{\text{в}}-\varphi_{\text{п}}-\frac{\pi}{4}\right)-\frac{1}{2}v_{\text{в}}+\frac{v_{\text{т}}}{\lambda_1}\frac{1}{(1+\mathrm{Pr}^{-1})^2}\left[2\mathrm{Pr}^{-1/2}\cos\left(\varphi_{\text{в}}-\varphi_{\text{т}}-\right.\right.$$

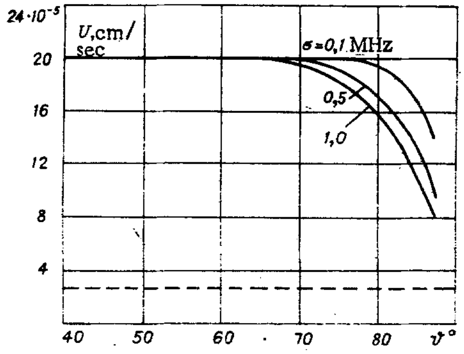

Fig. 1

$$-\frac{3}{4}\pi\Big) - (\mathrm{Pr}^{-1} - 1)\sin\Big(\varphi_{\text{в}} - \varphi_{\text{т}} - \frac{3}{4}\pi\Big)\Big]\Big\}; \tag{6}$$

the mass flow rate is given by the expressions [7]

$$\mathbf{U} = \mathbf{V}_2 + \mathbf{V}_{tr}, \ \mathbf{V}_{tr} = \langle(\xi_1 \cdot \nabla)\mathbf{V}_1\rangle,$$

in which ξ_1 is the displacement vector of the fluid particles in the acoustic wave, $\xi_1 = \int \mathbf{V}_1\,dt$. Using expressions (2) for $\mathbf{V}_1$, we obtain the following for $\mathbf{V}_{tr}$ on the outer side of the boundary layer:

$$\mathbf{V}_{tr} = V_{tr}(z)\,\mathbf{e}_x; \quad V_{tr} = A^2[1 + 2v_{\text{п}}\cos\varphi_{\text{п}} + v_{\text{п}}^2]. \tag{7}$$

Expressions (6) and (7) therefore enable us to calculate the acoustic streaming velocity for arbitrary grazing angles of the incident wave. We have carried out a numerical computation for streaming in air; the results are given in Fig. 1, in which it is seen that the flow velocity changes in the vicinity of the critical angle of Konstantinov. The general behavior of the velocity with variation of the frequency coincides with the behavior of the coefficient of reflection of acoustic waves from a perfectly rigid wall [8]; the higher the frequency the higher the rate of change of velocity. The dashed line represents the velocity U calculated according to expressions in [3, 5]. It follows from the graph that the acoustic streaming velocity for steep oblique wave incidence is much greater than the streaming velocity for grazing incidence. This conclusion is supported by published experimental investigations. For example, it has been shown [9] that if the acoustic wave is incident at a steep angle with the boundary, the mass-transfer process in the acoustic field, being determined mainly by the acoustic streaming, proceeds much more rapidly than for a boundary parallel to the direction of wave propagation.

We now find an analytical expression for the acoustic streaming velocity in the case $\eta \gg \Phi$. Here the viscosity and thermal conductivity of the fluid do not affect the reflection coefficients:

$$v_{\text{п}} = 1, \ \varphi_{\text{п}} = 0, \ v_{\text{в}} = \frac{2k_{11}}{\beta}, \ \varphi_{\text{в}} = -\frac{\pi}{4},$$

$$v_{\text{т}} = (\gamma - 1)\frac{k_{11}^2}{\alpha^2}; \quad \varphi_{\text{т}} = -\frac{\pi}{4},$$

and from (6) and (7) we obtain

$$V_2 = -\frac{4A^2}{c}\left[(1 - \sqrt{2}) - \sqrt{2}\frac{(\gamma - 1)}{\mathrm{Pr}(\mathrm{Pr} + 1)}\right]; \tag{8}$$

$$V_{tr} = \frac{4A^2}{c}. \tag{9}$$

It follows from (8) and (9) that V_2 and V_{tr} do not depend either on the grazing angle η or on the acoustic frequency, whereas for grazing angles near the critical angle of Konstantinov the streaming velocity is a function of these variables.

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MICROCONVECTIVE HEAT-AND MASS-TRANSFER PROCESSES IN FLUIDS WITH INTERNAL ROTATION

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A number of experimentally observed phenomena (the magnetoviscosity effect, i.e., increase of the viscosity of a ferromagnetic suspension in a magnetic field [1], and the entrainment of a polar fluid by a nonsteady magnetic field [2-4]) can be explained on the basis of the notion of internal rotations and the associated internal friction as a mechanism of momentum transfer from the field to the medium [5-8]. In line with the expanding study of the influence of internal rotations on macroscopic fluid motion there is also considerable interest in the development of mathematical models of asymmetric polarizable and magnetizable media [5, 9-12].

In the present article we show that the influence of internal rotations under definite conditions not only leads to a modification of the momentum-transfer law, but also proves significant in heat-transfer processes and, in the case of multicomponent fluids, mass-transfer processes as well, giving rise to a highly specific "microconvective" transfer mechanism.

Inasmuch as the significance of the internal-rotation concept is particularly highlighted in the case of suspensions and colloidal solutions, we discuss a certain volume of a suspension in a system S', in which macroscopic motion does not take place. This system rotates relative to the laboratory frame S with an angular velocity $\boldsymbol{\Omega} = (1/2)\mathrm{rot}\mathbf{v}$ (rot = curl). In the system S' the particles of the suspension rotate with a velocity $\mathbf{R} = \boldsymbol{\omega} - \boldsymbol{\Omega}$, where $\boldsymbol{\omega}$ is their rotational velocity in the system S. The rotating particles together with the fluid entrained by them through viscosity induce a local microconvective heat transfer in the system S' in the case of a nonuniform temperature distribution in the fluid. When the distance between the particles is commensurate with their sizes and the latter are large, a possible outcome of the interaction of the temperature fields of the individual microvortices and heat transfer between them is a macroscopic heat flux q_r, which competes with the conductive heat flux q_0.

We estimate the ratio q_r/q_0 on the basis of the heat-transfer equation $v\nabla T = \varkappa\nabla^2 T$, applying it to the individual microvortex, in which case it is necessary to adopt as the characteristic space scale the microvortex radius l_0. Then $v \simeq R l_0$, and:

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